where c is the separation constant and

$$(\dot{\ }) = \frac{\mathrm{d}t}{\mathrm{d}t}(\ ) \qquad (\ )' = \frac{\mathrm{d}}{\mathrm{d}x}(\ )$$

If the separation constant is set to $c=n(n+1)$, the frequency expression as given by Mr. MacNeal results,

$$\omega_n = \omega_c \sqrt{3n(n+1)/2} \tag{6}$$

With $c=n(n+1)$, Eq. (5) has the form of Legendre's differential equation[3] and for $n=1$ the form of the solution is $Z=(\text{const.})x$, i.e. it is satisfied by the pitch (rigid-body) rotational mode. For other values of c, the resulting differential equation may not be satisfied by such a solution.

In Mr. MacNeal's model of the beam it is assumed that the beam has had zero bending stiffness, whereas in the beam model presented in the Note[1,4] it is assumed that the resistance of the beam to transverse loads is due to internal bending moments and the restoring effect of the axial tension due to gravity-gradient is negligible. Thus, the different conclusions reached by Mr. MacNeal and the authors, regarding the lowest vibrational frequencies, are due to the different models used to describe the motion of a thin flexible beam in orbit. In our opinion, a conclusive statement regarding the validity of either model in the limiting situation as the beam becomes slender can only be reached after appropriate in-orbit testing of the behavior of such a structure under (near) zero-gravity conditions. It would appear that as long as the beam offers any bending resistance to transverse loading that the model of Ref. 1 would be valid.

Regarding Mr. MacNeal's comments about the frequencies of parametric excitation, the authors do agree that $\omega_n/\omega_c \equiv 1.0$ is *not* a frequency of parametric excitation. At the center of the first unstable region of the Mathieu chart ($\delta=1.0$, $\epsilon=0$), $\omega_n/\omega_c=\sqrt{3}/2$. In Ref. 1 it was the intention of the authors to demonstrate the possibility of instability at very low natural frequency due to small-amplitude pitch motion. It appears that a number of points may be located in the first unstable region for $0<c<0.2$, and $\omega_n/\omega_c=\mathcal{O}(1)$. Figure 2 of Ref. 1 presents a clear example of such an instability. In the tensioned-string model proposed by Mr. MacNeal, a lower limit on the frequency is $\omega_n=3\omega_c>\sqrt{3}\omega_c/2$. Here again, the difference in results is apparently due to the different models and associated lower bound on the frequencies. If the analysis of Ref. 1 is extended to consider higher order resonances, care would have to be taken to include the correspondingly higher order nonlinear terms in Eqs. (5) and (6).[1] The Mathieu equation (10) is, at best, only an approximation to such a higher order nonlinear system.

References

[1] Kumar, V.K. and Bainum, P.M., "Dynamics of a Flexible Body in Orbit," *Journal of Guidance and Control,* Vol. 3, Jan.-Feb. 1980, pp. 90-92.

[2] Meirovitch, L., *Analytical Methods in Vibrations,* The MacMillan Co., New York, 1967, pp. 161-166.

[3] Spiegel, M.R., *Mathematical Handbook,* Schaum's Outline Series, McGraw Hill, New York, p. 146.

[4] Kumar, V.K. and Bainum, P.M., "Dynamics of a Flexible Body in Orbit," AIAA Paper 78-1418, AIAA/AAS Astrodynamics Conference, Palo Alto, Calif., Aug. 7-9, 1978.

Errata

G-80-069 refers to

G80-035 (P)

Optimal Continuous Torque Attitude Maneuvers

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THE order of the figures on pages 215 and 216 is incorrect. Figures 2a and 2b should be interchanged with Figs. 3a and 3b.

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